

Midterm Exam Mathematical Physics, Prof. G. Palasantzas

- Date 19-05-2016
- Total number of points 100
- 10 points free for coming to the midterm exam
- For all problems, justify your answer



Problem 1 (20 points)

Prove that $\lim_{n \rightarrow +\infty} \left(1 + \frac{3}{n}\right)^{5n} = e^{15}$

Problem 2 (20 points)


Determine whether the series $\sum_{n=1}^{+\infty} \frac{\cos(4n)}{1+5^n}$ is convergent or divergent.

Problem 3 (20 points)

Consider the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n4^n}$

- (a: 10 points) For which x is the series absolutely convergent ?
(b: 5 points) For which x is the series conditionally convergent ?
(c: 5 points) For which x is the series divergent?

Problem 4 (30 points)

 consider a spring with mass m , spring constant k , and damping constant $c = 0$, and let $\omega = \sqrt{k/m}$. If an external force $F(t) = F_0 \cos \omega t$ is applied (the applied frequency equals the natural frequency), use the method of undetermined coefficients to show that the motion of the mass is given by

$$x(t) = c_1 \cos \omega t + c_2 \sin \omega t + \frac{F_0}{2m\omega} t \sin \omega t$$

Tip: Consider the equation of motion $m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F(t)$ and set $c=0$